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On Consistent Boundary Conditions for $c = 1$ String Theory

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We introduce a new parametrisation for the Fermi sea of the $c = 1$ matrix model. This leads to a simple derivation of the scattering matrix, and a calculation of boundary corrections in the corresponding $1 + 1$–dimensional string theory. The new parametrisation involves relativistic chiral fields, rather than the non-relativistic fields of the usual formulations. The calculation of the boundary corrections, following recent work of Polchinski, allows us to place restrictions on the boundary conditions in the matrix model. We provide a consistent set of boundary conditions, but believe that they need to be supplemented by some more subtle relationship between the space-time and matrix model. Inspired by these boundary conditions, some thoughts on the black hole in $c = 1$ string theory are presented.

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1. Introduction

One of the current outstanding problems in theoretical physics is the detailed understanding of quantum processes that involve black holes. It is intriguing that we have a potential laboratory for studying these processes, the $c = 1$ matrix model, but frustrating that since the discovery of this model no significant progress has been made in studying its black hole physics.

We present here an alternative picture, to those generally discussed [1], of the relationship between the free fermions of the matrix model and the space-time tachyon of two-dimensional string theory. As the fields we use to describe the matrix model are relativistic they provide a more direct relationship between the matrix model and the relativistic spacetime physics. In spirit, our parametrisation is closest to that discussed in [2]. We rely for our intuition on the transform between the matrix model and the string theory recently discussed by Polchinski and Natsuume [3][1].

2. Fermi Sea in Light Cone Co-ordinates

The fermions of the $c = 1$ matrix model arise from diagonalising the matrix of the matrix model, taking a large N (dimension of the matrix) limit and finding the critical scaling of coupling with N such that in this limit the Feynman diagrams of the matrix model can be thought of as smooth Riemann surfaces of differing genus. This continuum limit can be described by the scattering problem for non-relativistic fermions in an inverted harmonic oscillator potential.

$$S = \tfrac{1}{2}\int (\dot{\lambda}^2 - \lambda^2)dx, \quad \dot{\lambda} = \frac{d\lambda}{dx} \tag{2.1}$$

The standard approach to this theory, at the classical level, is to look at the low energy excitations of the fermi sea. These bosonic excitations, the collective field, lead to a non-linear relationship between left and right moving excitations - the classical S-matrix. We will follow a similar approach, but the collective fields we will use are different as we will now describe.

[1] This transformation is motivated by comparing calculations in [4,5] and [6], but the explicit mapping between field equations in the matrix model and in the string theory was not discussed until the above mentioned paper by Polchinski and Natsuume.

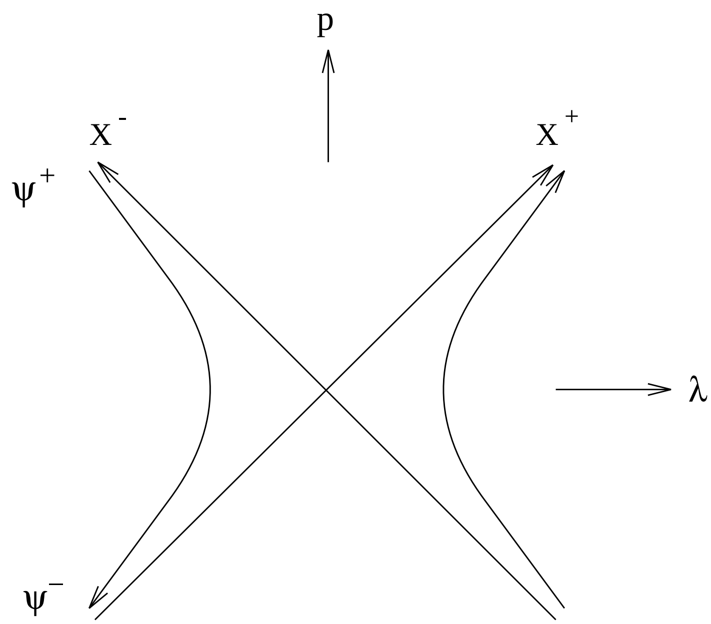

Fig. 1: Co-ordinate systems used throughout the paper and conventions for ingoing and outgoing fermion wavefunctions. Curved arrows show Hamiltonian flow on phase space.

With $\dot{\lambda} = p$ let $X^{\pm} = p \pm \lambda$, (see fig. 1 for our conventions). The classical equation of motion is $\dot{p} = \lambda$ and the general solution is $X^{\pm} = a_{\pm}(\sigma)e^{\pm t}$. For comparison with the parametrisation of the Fermi sea in terms of $p_{\pm}(\sigma, t)$ [5], we will present a simple derivation of the classical S-matrix for low energy scattering. Assuming the ingoing disturbance of the Fermi sea never crosses the potential barrier, (which is sufficient for this classical calculation), we can write, $a_{\pm}(\sigma) = \pm a(\sigma)e^{\pm b(\sigma)} = \pm a(\sigma)e^{\pm\sigma}$, and $h_{\pm}(\sigma) = log\ a(\sigma) \pm \sigma$.

For Fermi seas which obey the restriction that $X^+(X^-)$ is monotonic, $h_{\pm}(\sigma)$ are invertible. Then

$$\begin{aligned} \chi_{\pm}(X^{\mp}, t) &\equiv (-p^2 + \lambda^2 - \mu) \\ &= (a^2(\sigma) - \mu) \\ &= a^2(h_{\mp}^{-1}(x^{\mp} \pm t)) - \mu \end{aligned} \tag{2.2}$$

where $x^{\pm} = log(\mp X^{\pm})$. So we see that $\chi(v)$ are chiral fields, and one can show that they trivially satisfy $\chi_+(h_+(v)) = \chi_-(h_-(v))$. With a little more work one can also show that $\chi_+(w) = \chi_-(-w + log(-\mu + \chi_+(w)))$. This is the functional relationship between the ingoing and outgoing collective field that leads to the classical S–matrix of Polchinski and of Moore and Plesser[4]. Putting,

$$\chi_{\pm} = \partial_{\mp}\bar{S}_{\pm}(u^{\mp}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{\sqrt{2}} \bar{\alpha}_{\pm}(\omega) e^{i\omega u^{\mp}}, \tag{2.3}$$

the recursive functional relationship between $\chi_\pm$ leads to a non-linear relationship between the modes $\bar{\alpha}_\pm$ which is the tree level collective field S–matrix.

If $\alpha_\pm$ are the creation and annihilation operators for the 1+1–tachyon, then comparing the calculations of Kutasov and DiFrancesco to the tree level collective field S–matrix [4,5], the appropriate relationship between $\alpha_\pm$ and $\bar{\alpha}_\pm$ is

$$\bar{\alpha}_\pm(\omega) = \alpha_\pm(\omega)(\frac{\pi}{2})^{\mp\frac{i\omega}{4}}\frac{\Gamma(\mp i\omega)}{\Gamma(\pm i\omega)}, \tag{2.4}$$

and combining this transformation with the collective field S–matrix gives the classical tachyon S–matrix.

Another attractive feature of this decomposition of the Fermi sea excitations is that there is no breakdown of the collective field due to formation of folded configurations as the sea evolves in time [7] which in the extreme case involves pulses propagating over the barrier. To see the conservation of folds, define a fold be a place at which the collective field can no longer be well defined (it may become multiple valued). In our variables, this would appear as a turning point in $X_\pm(\sigma,t)e^{\mp t}$, as a function of σ. As these two functions are functions of σ only, the number of folds is encoded in $a(\sigma)$ alone and is clearly conserved.

3. Canonical transformations of the Fermi Sea

Even though this "light cone" description of the collective excitations of the Fermi sea is very simple as described in the previous section, to calculate higher order corrections to this picture it is by far most convenient to go back to the free fermion picture of the $c = 1$ matrix model. In this picture the description of scattering is obtained by a combination of fermionisation, free-fermi scattering and bosonisation [8]. The Feynman rules for the free fermi scattering consist of a wall vertex given by the non-relativistic reflection coefficient $R(x)$, and a free fermi propogator. Higher loop corrections are given by sums of ring diagrams and are discussed in detail in [8].

The same calculations may be carried out here, but first we consider a simple method to derive the scattering coefficient $R(x)$, the canonical transformation that relates ψ_+ to ψ_- (the ingoing and outgoing fermion wavefunctions). Again this is a globally defined transformation and thus we do not have to look at asymptotics to work out the relationship between the wall scattering and the relativistic fields at infinity (compare [4,5]).

As the coordinates (X^+, X^-) are canonically conjugate, the non-relativistic wavefunctions, ψ_+ and ψ_-, are the Fourier transforms of each other, (this is exactly as happens in non-relativistic quantum mechanics where the Fourier transform relates the position and momentum representations of a wavefunction).

$$\psi_-(X^+) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dX^- \; e^{iX^+X^-} \psi_+(X^-) \tag{3.1}$$

The Schrödinger equation becomes $\frac{1}{4}(X^+X^- + X^-X^+)\psi_\pm = (i\partial_t - \mu)\psi_\pm$. The normal ordering is the unique choice that ensures that both left and right moving wavefunctions are delta function normalisable.

For ψ_+ we find $(\frac{i}{2} - iX^-\partial_- + \mu)\psi_+^k(X^-) = k\psi_+^k(X^-)$. The wavefunctions are,

$$\psi_+^k(X^-) = a_k(X^-)^{-i(k-\mu)-\frac{1}{2}}\theta(X^-) + b_k(-X^-)^{-i(k-\mu)-\frac{1}{2}}\theta(-X^-) \tag{3.2}$$

and similarly for $\psi_-^k(X^+)$.

Applying the canonical transform to $\psi_+^k(X^-)$ we find

$$\psi_-^k(X^+) = \begin{cases} \frac{\Gamma(\frac{1}{2}-i(k-\mu))}{\sqrt{2\pi}(X^+)^{\frac{1}{2}-i(k-\mu)}}(a_k e^{i\frac{\pi}{2}(\frac{1}{2}-i(k-\mu))} + b_k e^{-i\frac{\pi}{2}(\frac{1}{2}-i(k-\mu))}) & X^+ > 0 \\ \frac{\Gamma(\frac{1}{2}-i(k-\mu))}{\sqrt{2\pi}(-X^+)^{\frac{1}{2}-i(k-\mu)}}(a_k e^{-i\frac{\pi}{2}(\frac{1}{2}-i(k-\mu))} + b_k e^{i\frac{\pi}{2}(\frac{1}{2}-i(k-\mu))}) & X^+ < 0 \end{cases} \tag{3.3}$$

From these expressions we see that the reflection coefficient is given by

$$R(k-\mu) = \frac{\Gamma(\frac{1}{2} - i(k-\mu))}{\sqrt{2\pi}}(e^{-i\frac{\pi}{2}(\frac{1}{2}-i(k-\mu))} + \frac{b_k}{a_k}e^{i\frac{\pi}{2}(\frac{1}{2}-i(k-\mu))}) \tag{3.4}$$

The boundary conditions are parametrised by the choice of $\frac{b_k}{a_k}$, ($a_k \neq 0$ for the scattering problems considered here). To relate these calculations to those with a boundary at a fixed value of λ, (as in [9,8]), let us consider in our framework the boundary conditions for which the left and right moving fermion wavefunctions have the same form. This amounts to requiring that

$$\frac{a_k e^{-i\frac{\pi}{2}(\frac{1}{2}-ix)} + b_k e^{i\frac{\pi}{2}(\frac{1}{2}-ix)}}{a_k e^{i\frac{\pi}{2}(\frac{1}{2}-ix)} + b_k e^{-i\frac{\pi}{2}(\frac{1}{2}-ix)}} = \frac{b_k}{a_k}. \tag{3.5}$$

It is easy to show that this implies $a_k = \pm b_k$.

For the case with $a_k = b_k$ we find

$$R_{I+}(x) = \sqrt{\frac{2}{\pi}}\Gamma(\tfrac{1}{2} - ix)cos(\frac{\pi}{2}(\tfrac{1}{2} - ix)) \tag{3.6}$$

and when $a_k = -b_k$

$$R_{I-}(x) = -i\sqrt{\frac{2}{\pi}}\Gamma(\tfrac{1}{2} - ix)sin(\frac{\pi}{2}(\tfrac{1}{2} - ix)). \tag{3.7}$$

The second (odd) case, is identical (to a phase) to the result of [8] for a wall at $\lambda = 0$.

For later purposes we will also write down the reflection coefficient for the no wall scattering in terms of $\psi_\pm$. This means there is no incoming wave from the right hand side of the barrier, or in other words $\psi_+ = 0$ for $X^- < 0$. Then

$$R_{II}(x) = \frac{1}{\sqrt{2\pi}}\Gamma(\tfrac{1}{2} - ix)e^{-i\frac{\pi}{2}(\frac{1}{2}-ix)} \tag{3.8}$$

Notice that the relativistic free fermions that we are describing, can be exactly bosonised. In the language of [10] the S–matrix is related to a bosonisation of fermionic Bogoliubov transformations on the in and out states of the Fermi sea. In the discussion of [8], the relativistic bosons that arise are found in the asymptotic behaviour of the collective field. In our description the bosons are everywhere relativistic. In the fermion field theory the S-matrix may be modified by choosing different in and out vacua around which one considers scattering. It would be intriguing to find some relationship between such choices of vacuum states for the fermions, choices of vacua for the bosons, and thus possibly to vacuum states for quantum fields in flat or curved spacetimes. From quantum field theory in curved space–times, we know that understanding vacuum states of the field involves understanding the relationship between wavefunctions in different asymptotic regions. It is thus necessary for us to understand the relationship between asymptotic regions in the matrix model and in the space–time. We will say a little more about this at the end of the next section, after we have discussed consistency of boundary conditions in the matrix model.

4. Low energy string theory

We can now easily write down the expression for the full quantum $1 \to n$ amplitudes of the matrix model collective field with general boundary conditions. One inserts the appropriate reflection coefficient as computed in the previous section, in the formulae derived in [8]. For example, the expression for the $1 \to 2$ amplitude is

$$\begin{aligned} \sqrt{3}\mu^{-i\omega} S(\omega;\omega_1,\omega_2) &= (\int_0^{\omega_1} - \int_{\omega_2}^{\omega})dx \ R(x-\mu)R(\mu+\omega-x) \\ &\equiv f_\mu(\omega,\omega_1) \end{aligned} \tag{4.1}$$

We can use $f_\mu(\omega,\omega_1)$, adapting a calculation of Polchinski and Natsuume [3], to derive the second order correction to the outgoing field in terms of the ingoing field. We will sketch the outline, (the full details are well explained in [3]).

$$
\begin{aligned}
S_-^{(2)}(x^+) &= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \, \frac{1}{\sqrt{2}i\omega} \int d\omega_1 f_\mu(\omega,\omega_1) e^{i\omega x^+} \frac{\Gamma(-i\omega)}{\Gamma(i\omega)} \frac{\Gamma(-i\omega_1)}{\Gamma(i\omega_1)} \frac{\Gamma(-i(\omega-\omega_1))}{\Gamma(i(\omega-\omega_1))} \\
&\sqrt{2}i\omega_1 \int dx_1^- \; e^{-i\omega_1 x_1^-} S_+(x_1^-) \sqrt{2}i(\omega-\omega_1) \int dx_2^- \; e^{-i(\omega-\omega_1)x_2^-} S_+(x_2^-) \\
&= \frac{e^{x^+}}{\pi^2\sqrt{2}} \int dx_1^- \; dx_2^- \; S_+(x_1^-) S_+(x_2^-) \int d\omega_1 \; f_\mu(-i,\omega_1) e^{-i\omega_1(x_1^- - x_2^-) - x_2^-}
\end{aligned} \tag{4.2}
$$

Here $x^\pm = t \pm \phi$, where (t,ϕ) are the string target space co-ordinates. The second equality was obtained from the first by considering the limit as $x^+ \to -\infty$ which is dominated by the first pole of the integrand in the lower half plane. The leading order of perturbation theory in $1/\mu$ at $\omega = -i$, gives us $f_\mu = \frac{1}{2\pi}$. The integral over ω_1 leads to a delta function in $x_1^- - x_2^-$ giving the tree level correction,

$$
S_-^{(2)tree}(x^+) = \frac{e^{x^+}}{2\pi^2\sqrt{2}} \int dx^- S_+(x^-)^2 e^{-x^-}. \tag{4.3}
$$

One can show that these expressions are the same as one derives from low energy string theory in the limit that $x^+ \to -\infty$, (again we refer the reader to [3] for more details). In string theory at $x^+ \to -\infty$ we are actually calculating in a weak tachyon perturbation theory, and not a large μ expansion as one does in the matrix model. One can see this by noticing that the expansion parameter in (4.3)is $e^{2\phi}\partial_x S$. For early enough times this quantity is always small regardless of the size of μ.

Therefore there is potentially some mixing of the non–perturbative physics of the matrix model, (where the perturbation theory is in terms of $1/\mu$), with the perturbative physics of the low energy string theory. Such would be a direct result of the non–local nature of the transformation between these two theories arising from the Γ/Γ transformation on the $\alpha(\omega)$ (2.4), (a Hankel transform in position space). For example, a Gaussian tachyon pulse maps to a collective field excitation that has a decaying exponential early time behaviour, $\bar{S}(x^-) \sim e^{x^-}$[3]. In the collective field of the matrix model, this means that part of this ingoing excitation of the fermi sea spends an exponentially long time near the quadratic turning point, (even though the bulk of the pulse is centered on $x^- \to x_0$). Thus the non–perturbative tunneling rate is enhanced. Such enhancement can cause corrections to the low energy effective action in the string theory. We now proceed to calculate these corrections.

4.1. Non-perturbative corrections to low-energy string theory.

We will evaluate

$$S_-(x^+) = \frac{e^{x^+}}{\pi\sqrt{2}} \int dx_1^- \; dx_2^- \; S_+(x_1^-)S_+(x_2^-) \int d\omega_1 f_\mu(-i,\omega_1) e^{-i\omega_1\Delta - x_2^-} \tag{4.4}$$

where Im $\omega_1 = -\frac{1}{2}$ and $\Delta = x_1^- - x_2^-$.

Notice that for R_{II}(3.8)and R_{I+}(3.6), the integrand in $f_\mu(-i,x)$ has a double pole at $x = \mu - \frac{i}{2}$, which is a point through which the ω_1 integration contour is required to pass. To investigate the effect of this we deform slightly away from $\omega = -i$ to $\omega = -i + i\epsilon$. The double pole is now a pair of poles at $x = \mu - \frac{i}{2}$ and $x = \mu - \frac{i}{2} + i\epsilon$. Possible subtleties can now be anticipated to arise from the choice of integration for the two contours that appear in f_μ. For the region of the ω_1 integral near the poles, (the part of the ω_1 integral near $\omega_1 = 0$ is already accounted for and gave rise to (4.3))

$$\int d\omega_1 f_\mu(-i,\omega_1) e^{-i\omega_1\Delta - x_2^-} = \int d\omega_1 e^{-i\omega_1\Delta} \left(\int_0^{\omega_1} - \int_{-i}^{-i-\omega_1}\right) \frac{dx}{(x-\mu+\frac{i}{2})(x-\mu+\frac{i}{2}-i\epsilon)} \tag{4.5}$$

and concentrating on the first term

$$\begin{aligned}
&\int d\omega_1 e^{-i\omega_1\Delta} \int_0^{\omega_1} \frac{dx}{(x-\mu+\frac{i}{2})(x-\mu+\frac{i}{2}-i\epsilon)} \\
&\qquad = e^{-i\mu\Delta - \frac{\Delta}{2}} \int_{-\infty}^{\infty} d\nu e^{-i\nu\Delta} \int_0^{\nu-\frac{i}{2}+\mu} \frac{dx}{(x-\mu+\frac{i}{2})(x-\mu+\frac{i}{2}-i\epsilon)} \\
&\qquad \sim ie^{-i\mu\Delta - \frac{\Delta}{2}} \Big(\int_{-\infty}^{\infty} d\nu e^{-i\nu\Delta} \int_\kappa^{\frac{\epsilon}{2}} \frac{dy}{(\nu+iy)(\nu+iy-i\epsilon)} \\
&\qquad - \frac{2\pi}{\epsilon} \int_0^{\infty} d\nu e^{-i\nu\Delta} \Big)
\end{aligned} \tag{4.6}$$

The approximation in the second line of this expression is obtained by restricting the x integration to the region near the poles. The additional term that appears in the last line of (4.6) arises from pinching the dx contours as $\epsilon \to 0$. When $\omega_1 > \mu - \frac{i}{2}$ and $\omega_1 < -\mu - \frac{i}{2}$ the contours are pinched between the two poles.

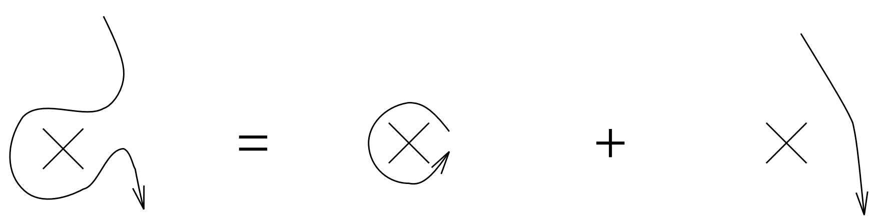

Fig. 2: Deformation of the x integration contour around the pole at $\mu - \frac{i}{2} + i\epsilon$ for $\omega_1 > \mu - \frac{i}{2}$.

The deformation of the contour required to navigate the poles for (4.6) is shown in fig. 2. By similar manipulations we find for the second term,

$$ie^{i\mu\Delta-\frac{\Delta}{2}}\left(\int_{-\infty}^{\infty}d\nu e^{i\nu\Delta}\int_{-\kappa}^{\frac{\epsilon}{2}}\frac{dy}{(\nu+iy)(\nu+iy-i\epsilon)}-\frac{2\pi}{\epsilon}\int_{0}^{\infty}d\nu e^{i\nu\Delta}\right). \tag{4.7}$$

We used the choice for all contours that one obtains by taking the integrals along the real axis and continuously deforming them into the lower half plane as $\omega\to -i$.

For this choice the final result has two terms on the right hand side that are important for the early time low energy string scattering. One term is that in (4.3) and agrees with the low energy string theory. The other, which comes from the residues of the pole shown in the first term on the right hand side of fig. 2, and a similar pole from the other part of the integral, is manifestly in disagreement with the low-energy string theory. It is,

$$\delta S_{-}^{(2)}(x^{+})=\frac{2\pi}{\epsilon}\int dx_1^-\ dx_2^-\ S_{+}^{(1)}(x_1^-)S_{+}^{(1)}(x_2^-)\frac{sin\mu\Delta}{\Delta}e^{-\frac{1}{2}(x_1^-+x_2^-)}, \tag{4.8}$$

where we still need to take the limit as $\epsilon\to 0$. This enables us to rule out this set of boundary conditions.

One may try alternative, though less natural, prescriptions for the paths of integration, and though one may remove the $\frac{1}{\epsilon}$ divergence, one cannot remove an additional finite non-local correction to the correct second order low energy string theory.

Of our three reflection coefficients R_{I+},R_{I-},R_{II}, only R_{I-} is free from corrections in the second order calculation, coming from double poles as discussed above. However, for $\omega=-3i$, which appears at higher orders in the weak field perturbation theory, one also finds double poles in $R_{I-}(x)R_{I-}(\omega-x)$, which the integration contour must negotiate. These will contribute again to the perturbation theory in a manner not in agreement with the known behaviour of the low-energy string theory[2], so R_{I-} must also be ruled inconsistent.

We have not discussed other choices for a_k/b_k, or other locations of the wall in $R(x,A)$. In both cases the pole structure is more complicated than in the simple symmetric cases we have discussed, and in general are probably more problematic. For example, in the $A\to\infty$ limit of $R(x,A)$ the poles in the lower half plane accumulate along the real axis. This will

[2] Although the tangle of contours that one needs to sort out is greater for $\omega=-3i$. We have calculated corrections for $\omega=-2i$, similar to those discussed above, in the perturbation expansion of $S_{-}^{(3)}$. Then only R_{II} has double poles that produce inconsistencies and $R_{I\pm}$ have single poles at $\omega=\mu-\frac{i}{2}$ that are potentially problematic. For $\omega=-3i$, $R_{I\pm}$ and R_{II} have several poles that will potentially produce additional corrections to the low energy string theory.

cause corrections to the above calculations in the first step, where one takes $x^+ \to -\infty$, as the first pole that one encounters in the lower half plane will not be at $\omega = -i$ but at $\mathrm{Im}\ \omega > -1$.

As the problems arise from the poles in $R(x)$ for $\mathrm{Im}\ x < 0$, we may attempt to eliminate them by simply requiring boundary conditions for which no such poles appear in the lower half plane. Referring to (3.4), we see that this requires $\frac{b_k}{a_k} = -e^{i\pi(\frac{1}{2}-i(k-\mu))}$,which gives,

$$R_{NS}(x) = -i\sqrt{\frac{2}{\pi}}\Gamma(\tfrac{1}{2} - ix)\ sin\pi(\tfrac{1}{2} - ix)e^{i\frac{\pi}{2}(\frac{1}{2}-ix)}. \tag{4.9}$$

We did not write down this reflection coefficient in our discussion of boundary conditions in section 3, as we wanted some symmetry between the ingoing and outgoing wavefunctions. For this $R(x)$ we find that $\psi_-(x^+) = 0$ for $X^+ > 0$. Using these boundary conditions is probably not the solution for which we are searching, due to the lack of symmetry between ingoing and outgoing wavefunctions. It is amusing to note that this is closely related to the no wall scattering coefficient (3.8) for which $\psi_+(x^-) = 0$ for $X^- < 0$. Furthermore, $R_{NS}(x)$ appears as the behaviour of $R_{I-}(x, A)$, as $A \to \infty$ at $\mathrm{Im}\ x < 0$ [8] (for $\mathrm{Im}\ x > 0$ $R_{I-} \to R_{II}$). This suggests that maybe some less local identification between matrix model phase space and space–time is required for a consistent low–energy string theory. Similar suggestions were made in [11].

The most naive realization of this modified mapping would be to use R_{II} for ingoing fermions, and R_{NS} for the outgoing fermions, or vice versa, in the formulae of [8]. This will cause problems in the low energy string theory as one may verify by calculating, similarly to the above, the contribution from the logarithmic branch cut in $f_\mu(-i, \omega_1)$, at $\omega_1 = \mu - \frac{i}{2}$, (the source of the contribution is again the residue of the pole that needs to be included due to the deformation of the x integration contour by the pole). Another resolution would simply be to use $lim_{A\to\infty} R_{I-}(x, A) = R'_{II}(x)$. $R'_{II}(x)$ is singular only in the sense that it is discontinuous across its branch cut along the real axis. More significant than potential complications arising from this branch cut, is the similarity between these reflection coefficients and choices for bases of wavefunctions of quantum fields in the presence of black holes. In such a case one may consider *in* states that involve no component crossing the past horizon (compare in R_{II} with $\psi_+(X^-) = 0$ for $X^- < 0$), and *out* states that involve no component crossing the future horizon (compare in R_{NS} with $\psi_-(X^+) = 0$ for $X^+ > 0$) [3]. Taking this idea seriously suggests that we should find Hawking radiation in the quantum tunneling through the inverted harmonic oscillator, enhanced by the Hankel transform that relates the matrix model and string theory [13].

[3] See for example [12], where a natural basis of vertex operators consists of $\{U^\lambda_\omega, V^\lambda_\omega\}$ where U^λ_ω vanish on the past horizon and V^λ_ω vanish on the future horizon.

5. Conclusions

The picture that we have developed here is unfortunately still considerably removed from the picture of string fields in space-time. To get the string amplitudes correct to tree level, it is known that one needs to multiply the collective field by a ratio of gamma functions, and that the space-time tachyon then is a Hankel transform of the collective field. This does not greatly enlighten us as to the relationship between the matrix model fields and the string fields. In particular, the main missing ingredient is the bulk scattering of tachyons. The scattering that comes from the reflection off the upside-down harmonic oscillator is so-called wall scattering of tachyons.

However we do believe that using the fields that we have introduced above, the relationship of the physics of the Fermi sea to space-time physics may be elucidated. The fields are manifestly relativistic and thus are good candidates for fields in the matrix model that have a simpler relationship to the vertex operators of the space-time string theory. It is intriguing that the decomposition implies a global structure of free field *in* states, and free-field *out* states with exact relativistic bosonisation. These in and out fields are related simply by a canonical transformation on the non-relativistic fermion wavefunctions when expressed in the light-cone bases. The difficult fields to find are the actual space-time string fields off-shell[4]. The relationship to the macroscopic loop operators of [14] is not clear physically, though one can use the technique of canonical transformations to write an expansion of the macroscopic loop operators in powers of the in or out collective field modes. The formula that one derives by this process is non–local and similar to the momentum space form of the tree level string amplitudes.

Our fields also have a strong resemblance to the in and out tachyon vertex operators that arise in the string theory description of this model [6]. In $1+1$-dimensional string theory the tachyon vertex operator has the form,

$$T_k^{\pm} = e^{ikx \pm (ik-\sqrt{2})\phi}, \tag{5.1}$$

where x is the flat direction and ϕ is the Liouville direction. This is suggestive of a relationship to the bosonisation of our chiral fermion oscillators, for which the wavefunctions are

$$\psi \sim e^{ikt - ikx^- - \frac{1}{2}x^-}. \tag{5.2}$$

[4] Although with a slightly different definition of the macroscopic loop operator one can find the ratio of gamma functions of (2.4) directly in the fourier components of this redefined operator. In the notation of Moore and Seiberg [14], we replace $Tr(e^{-lM})$ with $Tr(J_1(e^{-l(\dot{M} \pm M)}))$.

In this paper we have found that our light–cone decomposition of the Fermi sea allows a simple discussion of boundary conditions, and helps us to isolate a consistent set of such. In discussing consistency conditions, Polchinski[11] reached similar conclusions to ours on the subject of the boundary conditions for $R_{I\pm}$, R_{II}, and $R(x, A)$, in that they all produce inconsistent space–time string physics. The details of his arguments and ours do not agree. For example, we do not find a key role played in our calculations by the conserved charges v_{mn}, whereas these charges form the basis for the conclusions in [11]. In the light cone formulation where no walls appear, charges are in general always conserved. Here the breakdown of the low energy string theory arises through an interplay between the unstable fermion–fermion resonances [8], where a fermion pair is stuck at the top of the inverted oscillator potential, and the Hankel transform.

The main challenges that lie ahead are to make explicit the exact nature of the consistent non–perturbative completion of the matrix model that we have presented, and to find more about relationships between these boundary conditions and the black hole. The resolution of the first appears to be related to an identification of the relationship between the asymptotic regions in the matrix model and string theory. Certainly an understanding of the black hole requires an understanding of asymptotic regions (and horizons also). Our results, we believe, represent progress in this direction.

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